Approximate Formula for Sine by Bhāskara I

भद्रायां सुमतौ यतेम | -ऋग्वेद (६. १. १०)
Let us strive for the wisdom that leads to the welfare of all

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Siddhāntic Literature

- Earliest cmprehensive literature available is a *Siddhāntic* text,

  Āryabhaṭīya of Āryabhaṭa (499 CE)

- Earliest Karaṇa text is

  Karaṇaratna of Devācārya (629 CE)

Siddhāntic texts

- *yuga-**ravi**-bhagaṇāḥ* *khyughṛ* *śaśī*

  cayagiyiṅuśuchlṛ ku ... . | . . . laṅkāyām ||

  (Āryabhaṭīya – 499 CE – Bihar)

- *arka**paryayāḥ* *kha-abhra-kha-gagana-netra-guṇa-sāgara* *| ... ...*

  *(Siddhāntaśiromaṇi -- Bhāskara II - 12*th *Century – Mahārashtra)*

- *nānājñānapragadbho**, tilabalamasusūkṣmam*

  *. . . . . . . **arkā**deḥ paryayāḥ syuḥ ||*

  (Karaṇapaddhati – 1532 CE; Kerala)

Bhūtasaṅkhyā system

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- The word Bhūtasaṅkhyā is a compound word which has two constituents, namely bhūta and saṅkhyā–referring to a 'being' and a 'number' respectively.

- Hence bhūtasaṅkhyā = number associated with the being

- For instance:

Words Employed in Sanskrit Astronomical texts

- Here words commonly employed in Sanskrit such as:

- The physical entities such as Earth, Moon, planets, ocean, mountain, fire, sky, direction etc.

- The parts of human body such as eyes, ears, knees, hands, etc.

- The animals such as serpent, horse, elephant etc.

Example

String --> *netra kha bhūmi śaśi sarpa*

Example

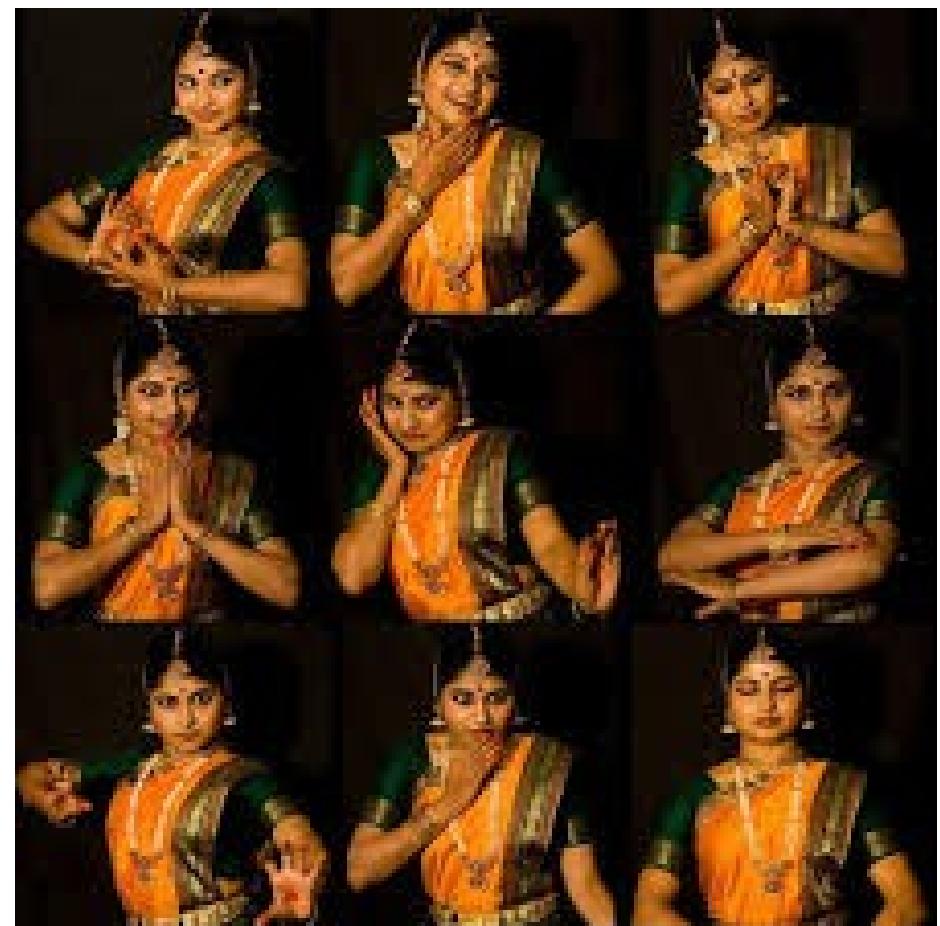**String -->** *netra kha bhūmi śaśi sarpa*

Number --> *2   0   1   1   8*

- **Number corresponds to the String**

  "81102"

What if two or more numbers associaed

rasa/रस

रस/*rasa*

- *yuga-*ravi*-bhagaṇāḥ* *khyughṛ* *śaśī*

  cayagiyiṅuśuchlṛ ku … …. | *. . . laṅkāyām* ||

  (Āryabhaṭīya – 499 CE – Bihar)

- arka*paryayāḥ* kha-abhra-kha-gagana-netra-guṇa-sāgara | … …

  (Siddhāntaśiromaṇi -- Bhāskara II - 12ᵗʰ Century – Mahārashtra)

- nānājñānapragadbho*, tilabalamasusūkṣmam*

  . . . . arkā*deḥ paryayāḥ syuḥ* ||

  (Karaṇapaddhati – 1532 CE; Kerala)

- *Kha-abhra-kha-gagana-netra-guṇa-sāgara*

- *4320000*

Āryabhaṭan system

- Based on the letters of Sanskrit Alphabets.
- Numbers are encoded in each letter.

1	2	3	4	5	6	7	8	9	10
क्/k	ख्/kh	ग्/g	घ्/gh	ङ्/ṅ	च्/c	छ्/ch	ज्/j	झ्/jh	ञ्/ñ

11	12	13	14	15	16	17	18	19	20
ट्/ṭ	ठ्/ṭh	ड्/ḍ	ढ्/ḍh	ण्/ṇ	त्/t	थ्/th	द्/d	ध्/dh	न्/n

21	22	23	24	25					
प्/p	फ्/ph	ब्/b	भ्/bh	म्/m	–	–	–	–	–

30	40	50	60	70	80	90	100	
य्/y	र्/r	ल्/l	व्/v	श्/ś	ष्/ṣ	स्/s	ह्/h	–

Number	10^0	10^2	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}	10^{16}
Vowels	a/ā	i/ī	u/ū	ṛ	ḷ	e	ai	o	au

Decoding the number

- Let us take the string ख्युघृ/*khyughṛ*

- *Split that into sub units* ख्यु *and* घृ

- Assign the numbers to the each letter

$$\text{ख्} = 2, \text{य्} = 30, \text{उ} = 10^4 ;$$

$$\text{घ्} = 4, \text{ऋ} = 10^6$$

$$(\text{ख्} + \text{य्}) \times \text{उ} + \text{घ्} \times \text{ऋ} = 4320000.$$

Kaṭapayādi System

- The name *Kaṭapayādi* stems the fact that here the sanskrit alphabets ka, ṭa, pa, ya etc. are used to denote the numbers
- According to the system,
    - the vowels standing alone, represent the number zero.
    - the same vowel in conjunction with the consonants have no numerical significance.

Number	1	2	3	4	5	6	7	8	9	0
Consonants	k	kh	g	gh	$\dot{n}$	c	ch	j	jh	$\bar{n}$
used	t	$ṭh$	$ḍ$	$ḍh$	$ṇ$	t	th	d	dh	n
to represent	p	ph	b	bh	m	–	–	–	–	–
numbers	y	r	l	v	$ś$	$ṣ$	s	h		–

Table : The *Kaṭapayādi* system of numeration.

Decoding the number

- Let us take the phrase

  nānājñānapragadbhaḥ

- Assign the numbers to the consonants

- Assign zero to stand alone vowel/s

  nā = 0; nā=0; jña=ñ=0; na = 0;

  pra=r=2; ga=3; dbha=bh=4.

- Reverse the order

- Number Corresponds to the phrase is

  "4320000"

मखि भखि फखि धखि णखि ञखि ङखि हस्झ स्ककि किष्ण श्घकि किघ्व घ्लकि किग्र हक्य धकि किच स्ग श्झ ड्व कृ प्त फ छ कलार्धज्याः ।।

मखि भखि फखि धखि णखि जखि ङखि हस्झ स्ककि किष्ण श्घकि किघ्व घ्लकि किग्र हक्य धकि किच स्ग श्झ ङ्व क्ऌ म फ छ कलार्धज्या: ।।

225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 199, 106, 93, 79, 65, 51, 37, 22, 7.

Rsine Table of Āryabhaṭa

प्रथमाच्चापज्याधार्धात् यैरूनं खण्डितं द्वितीयार्धम् ।

तत्प्रथमचापज्याधार्धांशैः तैस्तैरूनानि शेषाणि ।

Rsine Table of Nīlakaṇṭha

विलिप्ता दशकोना स्यात् राश्यष्टांशधनुः कलाः ।

आद्यज्याार्धात् ततो भक्ते सार्धदेवाश्विभिस्ततः । ।

त्यक्ते द्वितीयखण्डज्या द्वितीयाज्या च तद्द्युतिः

ततस्तेनैव हारेण लब्धं शोध्यं द्वितीयतः

खण्डात् तृतीयखण्डज्या द्वितीयस्तद्द्युतो गुणः

तृतीयः स्यात् ततश्चैवं चतुर्थाद्याः क्रमाद् गुणाः

Bhāskara I

मख्यादिरहितं कर्म वक्ष्यते तत्समासतः ।
चक्राधार्धांशकसमूहाद्द्विशोध्या ये भुजांशकाः ॥
तत्छेषगुणिता द्विष्ठा: शोध्याः खाभ्रेषुखाब्धितः ।
चतुर्थांशेन शेषस्य द्विष्ठमन्त्यफलाहतम् ॥
बाहुकोट्योः फलं कृत्स्नं क्रमोत्क्रमगुणस्य वा ।

लभ्यते चन्द्रतीक्ष्णांश्वोस्ताराणां वापि तत्त्वतः ॥

$$R\sin\theta = \frac{R\theta\left(180^0-\theta\right)}{\dfrac{1}{4}\left[40500-\theta\left(180^0-\theta\right)\right]} = \frac{4\,R\theta\left(180^0-\theta\right)}{\left[40500-\theta\left(180^0-\theta\right)\right]}$$

Mathematical Principles expressed through Poetic verses in Sanskrit

धन्यवाद!

In the end, we will conserve only what we love; we will love only what we understand, and we will understand only what we are taught.

--Baba Dioum, Senegalese Forest Engineer (1968, New Delhi)

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